PROSPECTS FOR THE USE of COMPLEX-VALUED LOGICS IN ARTIFICIAL INTELLIGENCE SYSTEMS

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Abstract
It is shown that the solution of the problem of the essence of intelligence (including human intelligence) is the key to the further development of artificial intelligence systems. Based on the consideration of human intelligence as a complexly structured system of information processing, it is shown that the criteria for distinguishing "machine" intelligence from "true" one, which go back to the Turing test, should be recognized as untenable. It is shown that it is more adequate to use such a criterion of manifestations of intelligence as the ability to lie. It is emphasized that this ability is not obligatory connect with negative connotation, as it is inextricably linked with the ability to display fantasy, and, consequently, to creativity. It is shown that the first step towards ensuring this criterion is the creation of artificial intelligence systems that operate with many-valued logics of various types. The possibilities of using logical variables represented by complex values (i.e. using the logical imaginary unit) are discussed. It is shown that the use of complex-valued logics makes it possible to pass to logics of a new type, the construction of which correlates with ancient Eastern philosophical systems. It is underlined that the apparatus of many-valued logics in many important cases can be reduced to algebraic systems. An analogue of the Algebraic normal form (Zhegalkin polynomial) for multi-valued logics corresponding to non-binary Galois fields is considered, which makes it possible to reduce any logical operations to algebraic ones.

Keywords
artificial intelligence, multivalued logic, Algebraic normal form (Zhegalkin polynomial), Galois fields, law of excluded middle, algebraic extension, essence of intelligence.

1. Introduction

In discussions involving the comparison of artificial intelligence (AI) with human intelligence, it is often not taken into account that human intelligence has actually gone through a very complex evolutionary path, during which its character has been transformed in one way or another. The most indicative in this regard are the works of Lévy-Bruhl, in particular [1], in which it was concluded that in primitive cultures there was no logical thinking at all, i.e. that apparatus of formal logic which goes back to Aristotle is a later invention. This remark is significant, at least, because in the discussions on the topic "can a machine think" that have been going on since the 60s of the last century, thinking, as a rule, is considered as something closely related to logic.

Moreover, in numerous historical studies, on which, in particular, the works of Mircea Eliade [2, 3] were based, it is shown that the myths formed by cultural peoples in antiquity (Ancient Greece, Ancient Egypt) should by no means be interpreted as some kind of fiction. Myth was (in many respects it still is [2, 3]) a means of reflection of reality. It was through myth, for example, in the era of classical Greece, that a person streamlined his vision of the surrounding reality. The myth regulated his life, gave integrity

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to society, etc. In fact, the myth performed the functions that religion, science and law later began to perform - all these areas of human activity were generated by the mythological picture of the world, which eventually ceased to satisfy the needs of society. It would not be a great exaggeration to say that religion, science and law have myth as their primary source. As Bertrand Russell [4] noted, even in classical mechanics one can see traces of animism, not to mention the fact that the fear of the punishment of the gods, which classical Greek myths speak of, de facto performed the functions that jurisprudence later began to perform.

Regulus, who in history became an example of loyalty to this word, most likely returned to a certain execution only because he swore by the gods in whom he believed.

These considerations, reflected in more detail in [5], make us take a significantly different look at the criteria that make it possible to distinguish between “true intelligence” and artificial intelligence, more precisely, AI that is built on principles that are ultimately based on mathematical logic, and, hence, the category of Truth, in the interpretation given by classical philosophy [6]. We emphasize that this is the basis of both what is called binary logic and the entire modern “digital world”, which part artificial intelligence systems still remain.

As the brief review presented above shows, the notion of truth was by no means inherent in human thinking from the very beginning - at least in the modern interpretation of this concept, but this does not mean at all that our rather distant ancestors did not have intelligence. The intellect of the overwhelming majority of human beings, strictly speaking, even now operates with completely different constructions (in particular, mythological ones [2,3]), to which truth has only an indirect relation. This is a sufficient reason to refuse to use such criteria as the Turing test and similar ones [7]. (Let us underline, that criticism of the applicability of the Turing test has been heard for quite a long time, and from different positions [8].)

We proceed from the fact [7] that the fundamental difference between human intelligence and conditionally “machine” intelligence (in the not quite definite sense in which this term was used and is used in discussions on the topic “is a machine capable of thinking”) is the ability to consciously and purposefully lie.

We emphasize that this statement does not provide for a mandatory negative connotation. In essence, it is the ability to lie that underlies what is called creativity. The man called Eugene Onegin never lived in the real Saint Petersburg. A.S. Pushkin’s roman, whose main character is Onegin, is fiction (like any other), i.e., from a purely formal point of view - a lie.

The same mechanisms of functioning of the intellect are responsible for both creativity and conscious deception. The construction of a mathematical model of a real physical process is the same act of creativity as writing a novel. A certain idealized construct, a product of fantasy, is generated. This picture only then is tested on correlations with reality, and the interpretation of the term “reality” in this sentence is more than variable [7]. So, in relation to literary creativity, it is customary to talk about artistic truth. A model of a physical phenomenon can be quite efficient even when it turns out that it has nothing to do with reality at all.

We also emphasize that a myth is, generally speaking, a kind of model of the world. Thus, the cosmogonic myths of ancient Greece - especially in the presentation of Hesiod (the poem "Theogony") - built a very consistent picture of the origin of things. We emphasize once again that the question of the conformity of the model of reality is far from unambiguous. Thus, many models of physical phenomena quite satisfactorily described the observed phenomena, although later they were recognized as having nothing to do with reality (the ether model, for example). Similarly, the mythological picture of the world, which is obviously a product of fantasy or creativity, was also (at a certain historical stage) quite efficient, at least from the point of view of ensuring the controllability of society.

Thus, the question "can a machine think?" should be reformulated - "can a machine deliberately lie?" [7].

Paradoxically, it is impossible to teach artificial intelligence to create without using some mechanisms similar to those that allow a person to lie.

Of course, it makes no sense to design a system that will purposefully deceive its developers, but the above theses, at least, clearly show how important (for the further development of artificial intelligence systems, particularly) is the ability to operate with representations that do not fit into the framework of the primitively understood opposition "Truth - Falsehood" [7].

In this paper, complex-valued logic is proposed as a tool that makes it possible to take at least the first step in the indicated direction.

2. Complex-valued logics and opposition "True - False"

Let us turn to the thesis called in [9] the Maltsev-Tarsky's thesis. It says that any description of a situation, which, from the point of view of a person, is complete, accurate and formal, can be represented as an algebraic system.

This thesis, as rightly noted in [9], has not been proven, but not refuted either. However, proceeding from it, one should try to take at least the first steps towards what was called "formalization of the process of conscious deception" in [7].

Such a step, from our point of view, is the development of formal logics, in which the very nature of the opposition of truth and falsehood changes the meaning.

Currently, there are a number of logical systems that do not imply the fulfillment of the law of the excluded middle [10,11], which is basic for classical logic (every statement is either true or false). In particular, in such logical systems, dating back to the works of Lukasiewicz [12], created under the influence of the success of non-Euclidean geometries, and his followers, along with the logical variables "Truth" and "False", one more variable is introduced, which is often interpreted as "Indefinitely". Accordingly, logic becomes, not binary, but ternary. Today, many-valued logics operate not only with three, but also with a large number of variables. Many significant results have been obtained in this direction [13,14], and the question of their use for creating artificial intelligence systems of various varieties is already clearly raised [15].

However, if we consider the basis of logics from a general methodological (philosophical) point of view [16,17], then it should be concluded that the nature of the opposition "True - False" in them de facto retained the Aristotelian interpretation. An alternative, as it is shown in [7], for example, are ancient Indian/Buddhist philosophical concepts, which asserted that truth cannot be expressed in words at all (truth lies outside "Yes" or "No").

In the VI-IV centuries, BC in India, the concept of "chatushkuotika" (i.e., "having four peaks") was developed, operating with four options for judging an object: it is, it is not-is, it is and is not-is, it is neither is and not-is [18]. In the future, it was significantly complicated, but for the purposes of this work, it will suffice to restrict ourselves to analogies with the simplest version.

A judgment of such a type as "an object both exists and does not exist at the same time" and a judgment conjugated to it in the sense of chatushkootika, in a formalized language, it is permissible to describe through the imaginary component of logical variables.

Accordingly with this approach, the list of logical values is significantly expanded; along with "Yes", "No", "Indefinitely" as in logical systems dating back to the logic of Lukasiewicz, the opposition "Yes" - "No" is supplemented by a pair on the imaginary axis, which represents the opposition "imaginary Yes" - "imaginary No ". One of the simplest interpretations of the last pair is, for example, as follows:

- It is true that the primary opposition implies "by default"
- It is false that the primary opposition implies "by default".

As applied to the apparatus of dialectical categories [17,19], such judgments allow an extremely transparent interpretation. An object/entity can be described in terms of a certain opposition, then the imaginary part of the logical variable takes on a positive value, regardless of which of the variants of the basic judgment is true, and vice versa, the object/entity is not described at all through such an opposition (simplifying, this question has nothing to do with it).

A complex Boolean variable can be represented in the usual form for complex numbers

$$a = a_1 + ia_2 \leftrightarrow (a_1, a_2) \qquad (1)$$

Consider one of the algebraic structures that can be used as the basis for constructing complex-valued logics. Classical logic, operating with the logical values "True" and "False", is known to be closely related to binary algebra, whose operations are reducible to operations on two elements 0 and 1 of the Galois field $GF(2)$.

Research in the field of many-valued logics, as a rule, operates with a tabular form of representation of logical operations (truth tables [14]). Already for logics operating with four possible values of logical variables, such a representation becomes very cumbersome. Therefore, it seems appropriate to start with the construction of the Galois field, which will play the same role for complex-valued logic as the field $GF(2)$. plays for classical logic.

Consider a set of sums of the form (1), in which the components a_1, a_2 can acquire values from the triple (-1,0,1), which corresponds to the Galois field $GF(3)$, i.e. ternary logic in the formulation of works [20,21].

We define the summation operation in accordance with the usual notation

$$a + b = a_1 + ia_2 + b_1 + ib_2 = (a_1 + b_1) + i(a_2 + b_2), \tag{2}$$

and the addition operations for the real and imaginary parts as

$$1 + 1 = -1;\ -1 - 1 = 1;\ -1 + 1 = 1 - 1 = 0, \tag{3}$$

$$i + i = -i;\ -i - i = i;\ -i + i = i - i = 0 \tag{4}$$

The multiplication rules remain the same as in the classical use of complex numbers, in particular,

$$-1 \cdot -1 = 1;\ i \cdot i = -1;\ i \cdot (a_1 + ia_2) = ia_1 - a_2 \tag{5}$$

By direct verification, one can easily show that for the considered set containing 9 elements (Table 1), endowed with properties (1) - (4), the field axioms are satisfied. Since all Galois fields with the same number of terms are isomorphic, it can be argued that the given set is a Galois field $GF(3^2)$.

For clarity, we write the relation for one of the special cases of calculations using the rules (1) - (4)

$$(1 \pm i) \cdot (1 \pm i) = 1 \pm i \pm i - 1 = \mp i, \tag{6}$$

which illustrates the closedness of the considered field with respect to the operations of addition and multiplication.

Table 1

Elements of the Galois field $G = GF(3^2)$ in the representation used.

a	a_2=-1	a_2=0	a_2=1
a_1=-1	a_11=-1-i	a_12=-1	a_13=-1+i
a_1=0	a_21=-i	a_22=0	a_23=i
a_1=1	a_31=1-i	a_32=1	a_33=1+i
a_1=-1	a_11=-1-i	a_12=-1	a_13=-1+i

Let us show that an arbitrary operation of the form

$$S = S(x, y);\ x, y \in G, \tag{7}$$

which can be interpreted as a logical expression, can be represented as a polynomial in x, y, which is an analogue of the Algebraic normal form (Zhegalkin polynomial) for the case under consideration.

Let us compose a polynomial in which the notation of Table 1 is used.

$$f(x) = \prod_{i,j=1}^{3} (x - a_{i,j}) \tag{8}$$

Using the identity,

$$(x - a)(x + a) = x^2 - a^2 \tag{9}$$

proved for the case of the field under consideration by direct verification, as well as equality (6), this polynomial can be reduced to the form

$$f(x) = x(x^2 - i)(x^2 + i)(x^2 - 1)(x^2 + 1) = x(x^4 + 1)(x^4 - 1) = x^9 - x \quad (10)$$

We emphasize that the integers 2, 4, etc. are not elements of the considered field, i.e. the corresponding degrees should be considered simply as an abbreviated form of notation

$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n} \quad (11)$$

We also emphasize that the result (10) is more than expected, since in finite commutative bodies all elements except zero are roots of the equation

$$f_0(x) = x^s - 1 = 0, \quad (12)$$

i.e. the right side of relation (10) could be written immediately, and the calculation is given only for clarity.

Using (10) it is also possible to construct polynomials of the form

$$f_{i,j}(x) = \frac{f(x)}{(x - a_{i,j})}, \quad (13)$$

for which, by construction,

$$f_{i_0,j_0}(a_{i,j}) = 0; i \neq i_0, j \neq j_0 \quad (14)$$

Using relation (14), one can write an analogue of the Algebraic normal form (Zhegalkin polynomial) for any function of arguments x, y, which is interpreted as a logical one.

We have

$$S(x, y) = \sum_{i_1,j_1,i_2,j_2=1}^{3} \frac{S_{i_1,j_1,i_2,j_2}}{f_{i_1,j_1}(a_{i_1,j_1}) f_{i_2,j_2}(a_{i_2,j_2})} f_{i_1,j_1}(x) f_{i_2,j_2}(y) \quad (15)$$

where the values S_{i_1,j_1,i_2,j_2} actually represent a "table" enumeration of the values that the function S(x,y) takes with the corresponding values of the arguments.

When substituting in (15) the specific values of the two arguments $= a_{i_{01},j_{01}}$, $y = a_{i_{02},j_{02}}$ to zero, according to the construction of functions (13), all terms are converted, except for the one that corresponds to a specific combination of values, which follows directly from (14).

$$S(a_{i_1,j_1}, a_{i_2,j_2}) = S_{i_1,j_1,i_2,j_2} \quad (16)$$

Thus, relation (15) makes it possible to implement an arbitrary function that takes values in the field $G = GF(3^2)$.

Thus, there is a fairly simple tool that allows one to reduce any operations of multivalued logics that allow representations in Galois fields to algebraic ones. This, in particular, makes it possible to abandon the use of truth tables, which, when moving to logics with a large number of variables, become very cumbersome. In particular, this tool can also be used when working with complex-valued logics that operate on an imaginary logical unit.

3. Conclusions

Thus, it is possible to reduce any constructions in terms of complex-valued logics to operations with polynomials given over the field $GF(3^2)$. This approach allows a natural generalization to functions interpreted as Booleans and depending on an arbitrary number of arguments. To do this, it suffices to write an analogue of expression (15) for the case of a larger number of arguments.

More broadly, the results of this work suggest that building non-standard logics (their multiplicity, realized at this stage in the development of the philosophy of logic and related disciplines, undoubtedly corresponds to the variety of ways of reasoning that an intellect worthy of being called that can use), one can move not by "algebraization logic", but also to go in the opposite direction, i.e. starting from some algebraic system, build a logical one. The existing variety of non-classical (non-Aristotelian) logics allows us to pose the question in this way, that is, using certain algebraic structures, for example, the well-studied Galois fields, it is permissible to give their elements (and operations on them) a logical meaning.

An example of the actual use of this approach is also presented in this paper. An algebraic Galois field is used, constructed by the method of algebraic extensions, in which a logical imaginary unit appears. Such a field, or rather the multi-valued logic corresponding to it, can be interpreted on the basis of comparison with ancient Eastern philosophical systems.

The constructed complex-valued logic can be considered as the first step towards the consistent formalization of the category of Falsehood (understood not as a primitive opposition to truth, but as the ability to reason about the non-existent, for example, to lie). This kind of approach seems promising, since, as shown in the work, it is the ability to deceive, inseparable from the ability to be creative, that is the basic criterion for distinguishing "true" intelligence from machine intelligence.

4. References

[1] Levy-Bruhl L. Le surnaturel et la nature dans la mentalité primitive // Presses universitaires de France. – 1963.

[2] Eliade M. The myth of the eternal return // Cosmos and history. Princeton University Press. – 2021. – Vol. 122.

[3] Eliade M. The sacred and the profane // The nature of religion. – Houghton Mifflin Harcourt. – 1959.

[4] Rassel B. Istoriya zapadnoj filosofii (History of Western Philosophy.). M.: Izd-vo AST. 2017. – 1024 s.

[5] Haziev V. S. O ponyatii «ob"ektivnaya istina» (On the concept of "objective truth" // Filosofiya i obshchestvo. (Philosophy and Society) – 2013. – №. 2 (70). – S. 47-64.

[6] Nechaev S. YU. Kitajskaya komnata Dzh. R. Serlya v kontekste problem filosofii iskusstvennogo intellekta (The Chinese room of J. R. Searl in the context of the problems of the philosophy of artificial intelligence) // Izvestiya Saratovskogo universiteta. Seriya Filosofiya. Psihologiya. Pedagogika. (Bulletin of the Saratov University. Series Philosophy. Psychology. Pedagogy.) – 2010. – T. 10. – №. 4. – S. 19-23.

[7] Gabrielyan O.A., Vitulyova Ye. S., Suleimenov I. E. Multi-valued logics as an advanced basis for artificial intelligence // Wisdom. – 2022. – accepted.

[8] Lucenko E. V. «Antit'yuring», ili kritika testa T'yuringa s pozicij informacionno-funkcional'noj teorii razvitiya tekhniki («Anti-Turing», or criticism of the Turing test from the standpoint of the information-functional theory of technology development) //Politematicheskij setevoj elektronnyj nauchnyj zhurnal Kubanskogo gosudarstvennogo agrarnogo universiteta. (Polythematic network electronic scientific journal of the Kuban State Agrarian University). – 2007. – №. 34. – S. 140-158.

[9] Palchunov D.E. Modeling of thinking and formalization of reflection. I: Model-theoretic formalization of ontology and reflection (Modeling of thinking and formalization of reflection. I: Model-theoretic formalization of ontology and reflection) // Philosophy of Science. - 2006. - No. 4 (31). - S. 86-114.

[10] Caret C. Hybridized paracomplete and paraconsistent logics. The Australasian Journal of Logic. – 2017 – 14(1).

[11] Abe J. M., Nakamatsu K., Silva Filho, J. I. Three decades of paraconsistent annotated logics: a review paper on some applications. Procedia Computer Science. – 2019. – 159. – P. 1175-1181.
[12] Bofill M., Manya F., Vidal A., Villaret M. New complexity results for Łukasiewicz logic. Soft Computing. – 2019. – 23(7) – P. 2187-2197.
[13] Kulik B. A. N-tuple algebra-based probabilistic logic. Journal of Computer and Systems Sciences International. 2007 – 46(1). – P. 111-120.
[14] Marcos J. On a Problem of da Costa // Essays on the Foundations of Mathematics and Logic 2 / Ed. by G. Sica. Polimetrica. – 2005. – P. 53-69.
[15] Suleimenov I. E., Matrassulova D. K., Moldakhan I., Vitulyova Y. S., Kabdushev S. B., Bakirov A. S. Distributed memory of neural networks and the problem of the intelligence's essence. Bulletin of Electrical Engineering and Informatics. 2022 – 11(1).
[16] Suleimenov, I.E., Vitulyova, Y.S., Bakirov, A.S., Gabrielyan, O.A. Artificial Intelligence: What is it? ACM International Conference Proceeding Series. – 2022. – P. 22-25. https://doi.org/10.1145/3397125.3397141
[17] Vitulyova Y.S., Bakirov A.S., Baipakbayeva S.T., Suleimenov I.E., Interpretation of the category of complex in terms of dialectical positivism. IOP Conference Series: Materials Science and Engineering. – 2020. – 946(1). – 012004. 10.1088/1757-899X/946/1/012004
[18] Maksimov D. YU. Logika N.A. Vasil'eva i mnogoznachnye logiki (Logic N.A. Vasiliev and many-valued logics) //Logicheskie issledovaniya. (Logical research). – 2016. – T. 22. – №. 1.
[19] Suleimenov, Massalimova, Bakirov, Gabrielyan. Neural Networks and the Philosophy of Dialectical Positivism. MATEC Web Conf. – 2018. – №214. – P. 02002 DOI: 10.1051/matecconf/201821402002
[20] Moldakhan I., Shaltikova D. B., Egemberdyeva Z. M., Suleimenov I. E. Application of ternary logic for digital signal processing. In IOP Conference Series: Materials Science and Engineering. – 2020. – Vol. 946. – No. 1. – p. 012002.
[21] Suleimenov I., Bakirov A., Moldakhan I. Formalization of Ternary Logic for Application to Digital Signal Processing. In Energy Management of Municipal Transportation Facilities and Transport. – 2019. – P. 26-35.